$A = D_{1,1}f(a), B = D_{1,2}f(a), C = D_{2,2}f(a)$, let $\Delta = \det\begin{bmatrix} A & B \\ C & D \end{bmatrix} = AC - B^2$. Prove that

(i) If $\Delta > 0$ and $A > 0$, f has a relative minimum at a.

(ii) If $\Delta > 0$ and $A < 0$, f has a relative maximum at a.

(iii) If $\Delta < 0$, f has a saddle point at a.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Levi theorem for step functions.

17. State and prove Riesz-Fischer theorem.

18. State and prove Weierstrass Approximation theorem.

19. Assume that one of the partial derivatives $D_1 f, \ldots\ldots D_n f$ exists of c and that the remaining $n-1$ partial derivatives exist in some n ball $B(c)$ and are continuous at c. Prove that f is differentiable at c.

20. State and prove implicit function theorem.

NOVEMBER/DECEMBER 2024

GMA22 — REAL ANALYSIS – II

Time : Three hours Maximum : 75 marks

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SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. When a property is used said to hold almost everywhere?

2. Define Lebesgue integrable.

3. Define measurable function.

4. When a function is called square integrable.

5. What is called Fourier Series of f ?

6. State the Reimann-Lebesque lemma.

7. Define total derivative of f at c?

8. What is called Jacobian matrix?

9. Define open mapping.

10. Define stationary point of f.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Let I be an interval which is the union of two subintervals say $I = I, UI_2$, where I_1 *and* I_2 have no interior points in common. If $f \in \cup(I)$ and if $f \geq 0$ *a.e.* on I, then prove that $f \in \cup(I_1)$, $f \in \cup(I_2)$ and $\int_I f = \int_{I_1} f + \int_{I_2} f$.

Or

(b) Let f be defined on I. If $f = 0$ almost everywhere on I, then prove that $f \in L(I)$ and $\int_I f = 0$.

12. (a) If f is Riemann-integrable on $[a, b]$ for every $b \geq a$, and assume there is a positive constant M such that $\int_a^b |f(x)|dx \leq M$, for every $b \geq a$, prove that f and $|f|$ are improper Riemann-integrable on $[a, \infty]$. Also prove that f is Lebesgue-integrable on $[a, +\infty]$ and the Lebesgue integral of f is equal to the improper Riemann integral of f.

Or

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(b) If $f \in L^2(I)$ and $g \in L^2(I)$, then prove that $f \circ g \in L(I)$ and $(af + bg) \in LH^2(I)$ for every real a and b.

13. (a) State and prove Parseval's formula.

Or

(b) State and prove the Riemann-Lebesgue lemma.

14. (a) Assume f is differentiable at c with total derivative T_c. Then prove that the directional derivative $f'(c;u)$ exists for every u in R^n and $T_c(u) = f'(c;u)$.

Or

(b) State and prove that Mean-Value theorem.

15. (a) If $f = u + iv$ is a complex-valued function with a derivative at a point z is C then prove that $J_f(z) = |f'(z)|^2$.

Or

(b) Let f be a real-valued function with continuous second-order partial derivatives at a stationary point a in R^2. Let